

**Supplementary Materials.**  
**Generalized species sampling priors with latent Beta  
reinforcements.**

Table 1: The table compares the Beta-GOS, HMM, and a Dirichlet Process model under different specifications of hyper-parameters when the data generating process is Beta-GOS( $\alpha_n = n, \beta_n = 1$ ) (1,000 replicates, 100 observations each)

Data Generating Process Beta-GOS ( $\alpha_n = n, \beta_n = 1$ )									
Simulation Method	Beta-GOS						HMM	DP	
	$\alpha_n = 10, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	$\alpha_n = 1, \beta_n = 6$			
$\tau = 0.25$									
Nclusters Ground Truth	5.09 ± 3.88	5.08 ± 3.88	5.08 ± 3.92	5.06 ± 3.88	5.05 ± 3.88	5.08 ± 3.88	5.18 ± 3.90	5.24 ± 3.88	
Nclusters Estimation	4.33 ± 2.77	4.14 ± 2.61	4.02 ± 2.55	3.89 ± 2.45	3.69 ± 2.28	3.60 ± 2.17	3.62 ± 0.83	4.51 ± 2.62	
Correct Assignment	0.97 ± 0.06	0.97 ± 0.06	0.97 ± 0.06	0.97 ± 0.06	0.96 ± 0.07	0.96 ± 0.07	0.91 ± 0.13	0.96 ± 0.08	
Predictive bias	3.72 ± 6.91	3.70 ± 6.90	3.65 ± 6.91	3.82 ± 6.95	3.94 ± 7.01	3.90 ± 6.94	4.04 ± 7.12	4.34 ± 7.27	
$\tau = 0.5$									
Nclusters Ground Truth	4.75 ± 4.12	4.74 ± 4.11	4.75 ± 4.13	4.75 ± 4.12	4.75 ± 4.12	4.75 ± 4.12	4.91 ± 3.67	5.08 ± 3.68	
Nclusters Estimation	3.67 ± 2.42	3.39 ± 2.18	3.27 ± 2.07	3.10 ± 1.95	2.87 ± 1.70	2.76 ± 1.54	3.66 ± 0.81	3.98 ± 2.23	
Correct Assignment	0.96 ± 0.08	0.96 ± 0.08	0.96 ± 0.08	0.95 ± 0.08	0.95 ± 0.09	0.94 ± 0.10	0.92 ± 0.12	0.94 ± 0.10	
Predictive bias	3.13 ± 5.89	3.17 ± 5.93	3.22 ± 6.04	3.38 ± 6.14	3.58 ± 6.34	3.63 ± 6.39	4.00 ± 6.86	4.28 ± 7.02	
$\tau = 1$									
Nclusters Ground Truth	4.98 ± 3.44	4.99 ± 3.43	4.98 ± 3.45	4.95 ± 3.43	4.97 ± 3.43	4.99 ± 3.43	5.24 ± 3.78	5.24 ± 3.77	
Nclusters Estimation	3.82 ± 2.10	3.28 ± 1.70	3.05 ± 1.53	2.75 ± 1.36	2.40 ± 1.09	2.23 ± 0.98	3.75 ± 0.70	3.28 ± 1.66	
Correct Assignment	0.92 ± 0.10	0.92 ± 0.10	0.92 ± 0.11	0.91 ± 0.11	0.89 ± 0.14	0.87 ± 0.16	0.89 ± 0.13	0.90 ± 0.14	
Predictive bias	4.29 ± 6.44	4.31 ± 6.45	4.38 ± 6.48	4.42 ± 6.62	4.56 ± 6.79	4.57 ± 6.86	4.90 ± 7.20	5.08 ± 7.15	

Table 2: The table compares the Beta-GOS, HMM, and a Dirichlet Process model under different specifications of hyper-parameters when the data generating process is Beta-GOS( $\alpha_n = 1, \beta_n = 1$ ). (1,000 replicates, 100 observations each)

Beta-GOS data ( $\alpha_n = \beta_n = 1$ )											
Simulation Method	Beta GOS						HMM			DP	
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 10, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	$\alpha_n = 1, \beta_n = 6$	4 States	5 States	6 States	$\theta = 1$
	$\tau = 0.25$										
Nclusters Ground Truth	2.04 ± 0.98	2.04 ± 0.98	2.04 ± 0.98	2.05 ± 0.98	2.04 ± 0.98	2.04 ± 0.98	2.04 ± 0.99	2.03 ± 1.05	2.01 ± 1.05	2.01 ± 1.06	2.03 ± 0.99
Nclusters Estimation	1.93 ± 0.94	2.06 ± 1.08	1.98 ± 0.97	1.95 ± 0.93	1.91 ± 0.89	1.89 ± 0.86	1.89 ± 0.85	3.35 ± 1.11	4.11 ± 1.44	4.90 ± 1.74	2.17 ± 1.13
Correct Assignment	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	1.00 ± 0.01
Predictive. bias	0.34 ± 1.08	0.28 ± 0.23	0.28 ± 0.21	0.28 ± 0.21	0.28 ± 0.21	0.28 ± 0.21	0.28 ± 0.21	0.67 ± 2.79	0.67 ± 2.75	0.64 ± 2.66	0.57 ± 2.23
	$\tau = 0.50$										
Nclusters Ground Truth	1.99 ± 0.93	1.98 ± 0.93	1.99 ± 0.93	1.97 ± 0.93	1.98 ± 0.93	1.98 ± 0.93	1.99 ± 0.93	2.03 ± 0.98	2.04 ± 0.99	2.04 ± 1.00	2.04 ± 0.97
Nclusters Estimation	1.90 ± 0.89	2.25 ± 1.29	2.05 ± 1.03	1.94 ± 0.93	1.86 ± 0.85	1.80 ± 0.78	1.79 ± 0.77	3.22 ± 1.17	4.01 ± 1.48	4.79 ± 1.78	2.27 ± 1.25
Correct Assignment	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	1.00 ± 0.01
Predictive. bias	0.62 ± 1.12	0.56 ± 0.45	0.56 ± 0.43	0.56 ± 0.43	0.56 ± 0.43	0.56 ± 0.43	0.56 ± 0.43	0.83 ± 2.31	0.85 ± 2.40	0.83 ± 2.30	0.85 ± 2.19
	$\tau = 1$										
Nclusters Ground Truth	2.04 ± 0.97	2.05 ± 0.97	2.04 ± 0.98	1.98 ± 0.92	1.99 ± 0.93	2.04 ± 0.97	2.04 ± 0.97	2.03 ± 0.98	2.04 ± 0.99	2.04 ± 1.00	2.04 ± 0.97
Nclusters Estimation	2.05 ± 1.14	2.45 ± 1.60	2.05 ± 1.04	1.95 ± 0.94	1.84 ± 0.82	1.79 ± 0.78	1.77 ± 0.76	3.22 ± 1.17	4.01 ± 1.48	4.79 ± 1.78	2.27 ± 1.25
Correct Assignment	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	1.00 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	1.00 ± 0.01
Predictive. bias	0.62 ± 1.12	0.56 ± 0.43	0.56 ± 0.43	0.55 ± 0.43	0.56 ± 0.43	0.56 ± 0.43	0.56 ± 0.44	0.83 ± 2.31	0.85 ± 2.40	0.83 ± 2.30	0.85 ± 2.19

Table 3: The table compares the Beta-GOS, HMM, and a Dirichlet Process model under different specifications of hyperparameters when the data generating process is Beta-GOS( $\alpha_n = 3, \beta_n = 1$ ). (1,000 replicates, 100 observations each)

Beta-GOS data( $\alpha_n = 3, \beta_n = 1$ )													
Simulation Method	Beta GOS										HMM		DP
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 10, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	$\alpha_n = 1, \beta_n = 6$	4 States	5 States	6 States			
	$\tau = 0.25$												
Nclusters Ground Truth	4.15 ± 1.81	4.12 ± 1.80	4.13 ± 1.82	4.15 ± 1.79	4.14 ± 1.81	4.16 ± 1.80	4.18 ± 1.79	4.18 ± 1.80	4.18 ± 1.79	4.19 ± 1.81	4.14 ± 1.81		
Nclusters Estimation	3.61 ± 1.49	3.79 ± 1.63	3.67 ± 1.51	3.63 ± 1.46	3.56 ± 1.42	3.50 ± 1.36	3.48 ± 1.31	3.46 ± 0.97	4.03 ± 1.37	4.62 ± 1.70	3.96 ± 1.72		
Correct Assignment	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.99 ± 0.01	0.97 ± 0.03	0.97 ± 0.04	0.97 ± 0.04	0.99 ± 0.02		
Predictive bias	0.67 ± 2.61	0.29 ± 0.29	0.29 ± 0.22	0.29 ± 0.22	0.29 ± 0.22	0.29 ± 0.22	0.28 ± 0.22	0.94 ± 3.43	1.10 ± 3.79	1.12 ± 3.82	1.29 ± 3.93		
	$\tau = 0.50$												
Nclusters Ground Truth	3.96 ± 1.67	3.99 ± 1.71	3.98 ± 1.71	4.00 ± 1.71	3.98 ± 1.70	3.98 ± 1.71	3.99 ± 1.70	3.99 ± 1.71	3.98 ± 1.71	3.99 ± 1.71	3.98 ± 1.70		
Nclusters Estimation	3.26 ± 1.31	3.59 ± 1.57	3.36 ± 1.35	3.27 ± 1.28	3.16 ± 1.19	3.04 ± 1.08	2.99 ± 1.04	3.52 ± 0.92	4.21 ± 1.27	4.88 ± 1.60	3.50 ± 1.55		
Correct Assignment	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.97 ± 0.03	0.97 ± 0.03	0.97 ± 0.03	0.98 ± 0.03		
Predictive bias	0.85 ± 2.09	0.57 ± 0.45	0.57 ± 0.43	0.57 ± 0.43	0.57 ± 0.43	0.56 ± 0.43	0.57 ± 0.43	1.08 ± 2.94	1.16 ± 3.15	1.17 ± 3.18	1.46 ± 3.36		
	$\tau = 1$												
Nclusters Ground Truth	3.97 ± 1.71	3.97 ± 1.71	3.98 ± 1.72	3.99 ± 1.72	3.99 ± 1.71	4.02 ± 1.71	4.02 ± 1.71	3.98 ± 1.71	4.00 ± 1.71	3.99 ± 1.71	3.99 ± 1.71		
Nclusters Estimation	2.82 ± 1.25	3.40 ± 1.71	3.04 ± 1.36	2.88 ± 1.21	2.69 ± 1.07	2.52 ± 0.92	2.43 ± 0.85	3.61 ± 0.85	4.41 ± 1.12	5.16 ± 1.43	2.99 ± 1.36		
Correct Assignment	0.97 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.98 ± 0.03	0.97 ± 0.04	0.97 ± 0.04	0.96 ± 0.04	0.97 ± 0.04	0.97 ± 0.04	0.97 ± 0.04		
Predictive bias	1.57 ± 2.86	1.17 ± 0.93	1.16 ± 0.89	1.16 ± 0.89	1.16 ± 0.90	1.17 ± 0.90	1.17 ± 0.90	1.49 ± 2.76	1.54 ± 2.88	1.56 ± 2.93	2.27 ± 4.12		

Table 4: The table compares the Beta-GOS and a Dirichlet Process model under different specifications of hyperparameters when the data generating process is a Gaussian Mixture of 5 Normal Components. (1,000 replicates, 100 observations each)

Mixture of 5 Gaussian distributions						
Simulation Method	Beta GOS					DP
	$\alpha_n = 10, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	$\alpha_n = 1, \beta_n = 6$	
$\tau = 0.25$						
Nclusters Ground Truth	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00
Nclusters Estimation	5.29 ± 1.24	4.99 ± 0.96	4.86 ± 0.85	4.55 ± 0.66	4.47 ± 0.64	5.54 ± 1.49
Correct Assignment	0.94 ± 0.08	0.94 ± 0.09	0.94 ± 0.08	0.92 ± 0.09	0.92 ± 0.09	0.93 ± 0.10
Predictive. bias	8.39 ± 8.89	8.35 ± 8.90	8.38 ± 8.89	8.36 ± 9.11	8.37 ± 9.18	8.82 ± 8.89
$\tau = 0.50$						
Nclusters Ground Truth	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00
Nclusters Estimation	5.22 ± 1.56	4.70 ± 1.21	4.48 ± 1.07	3.92 ± 0.82	3.80 ± 0.78	5.30 ± 1.91
Correct Assignment	0.87 ± 0.10	0.87 ± 0.11	0.86 ± 0.11	0.83 ± 0.12	0.82 ± 0.12	0.86 ± 0.13
Predictive. bias	8.77 ± 8.42	8.74 ± 8.36	8.44 ± 7.99	8.50 ± 7.84	8.43 ± 7.75	8.55 ± 8.53
$\tau = 1$						
Nclusters Ground Truth	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00	5.00 ± 0.00
Nclusters Estimation	4.96 ± 1.81	4.26 ± 1.35	3.92 ± 1.17	3.01 ± 0.86	2.81 ± 0.79	4.34 ± 1.59
Correct Assignment	0.77 ± 0.12	0.76 ± 0.12	0.75 ± 0.12	0.67 ± 0.13	0.64 ± 0.12	0.74 ± 0.14
Predictive. bias	8.56 ± 8.79	8.56 ± 8.83	8.55 ± 8.79	8.86 ± 9.01	8.73 ± 9.11	8.97 ± 8.55

Table 5: The table compares the Beta-GOS, HMM, and a Dirichlet Process model under different specifications of hyper-parameters when the data generating process is a Hidden Semi-Markov Model with 4 and 5 States, mean 15 and overdispersion 100. (1,000 replicates, 100 observations each)

i) Data Generating Process: Hidden Semi Markov Model (HSMM) with 4 states and NegBin(15, 0.15)									
Simulation Method	Beta GOS			HMM			DP		
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	3 States	4 States	5 States	$\theta = 1$
	$\tau = 1$								
Nclusters Ground Truth	3.67 ± 0.49	3.68 ± 0.49	3.67 ± 0.49	3.67 ± 0.49	3.67 ± 0.49	3.68 ± 0.49	3.69 ± 0.49	3.68 ± 0.49	3.68 ± 0.48
Nclusters Estimation	3.45 ± 1.15	3.45 ± 1.11	3.33 ± 0.98	3.18 ± 0.85	2.87 ± 0.74	2.99 ± 0.14	3.85 ± 0.50	4.62 ± 0.89	3.85 ± 1.51
Correct Assignment	0.85 ± 0.14	0.87 ± 0.13	0.87 ± 0.13	0.86 ± 0.14	0.80 ± 0.15	0.82 ± 0.12	0.88 ± 0.13	0.88 ± 0.15	0.82 ± 0.15
ii) Data Generating Process: Hidden Semi Markov Model (HSMM) with 5 states and NegBin(15, 0.15)									
Simulation Method	Beta GOS			HMM			DP		
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	4 States	4 States	5 States	$\theta = 1$
	$\tau = 0.5$								
Nclusters Ground Truth	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10	4.99 ± 0.10
Nclusters Estimation	4.74 ± 1.18	4.67 ± 1.11	4.52 ± 0.94	4.31 ± 0.92	4.02 ± 0.83	3.92 ± 0.39	5.21 ± 1.81		
Correct Assignment	0.88 ± 0.11	0.88 ± 0.11	0.89 ± 0.10	0.87 ± 0.12	0.83 ± 0.13	0.78 ± 0.11	0.85 ± 0.13		
	$\tau = 1$								
Nclusters Ground Truth	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21	4.95 ± 0.21
Nclusters Estimation	4.30 ± 1.34	4.22 ± 1.28	4.02 ± 1.10	3.68 ± 0.98	3.32 ± 0.83	3.93 ± 0.36	4.31 ± 1.55		
Correct Assignment	0.78 ± 0.12	0.79 ± 0.12	0.78 ± 0.13	0.76 ± 0.13	0.71 ± 0.13	0.77 ± 0.11	0.72 ± 0.15		

Table 6: The table compares the Beta-GOS, HMM, and a Dirichlet Process model under different specifications of hyper-parameters when the data generating process is a Hidden Semi-Markov Model with 4 and 5 States, mean 5 and overdispersion 50.

i) Data Generating Process: Hidden Semi Markov Model (HSMM) with 4 states and NegBin(5, 0.10)									
Simulation Method	Beta GOS				HMM			DP	
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	3 States	4 States	5 States	$\theta = 1$
	$\tau = 1$								
Nclusters Ground Truth	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07	4.00 ± 0.07
Nclusters Estimation	3.75 ± 1.26	4.07 ± 1.40	3.51 ± 1.12	3.22 ± 0.91	2.85 ± 0.74	2.99 ± 0.10	3.92 ± 0.39	4.76 ± 0.74	3.90 ± 1.56
Correct Assignment	0.80 ± 0.14	0.82 ± 0.14	0.81 ± 0.14	0.78 ± 0.15	0.73 ± 0.15	0.75 ± 0.10	0.82 ± 0.13	0.84 ± 0.14	0.77 ± 0.16
ii) Data Generating Process: Hidden Semi Markov Model (HSMM) with 5 states and NegBin(5, 0.10)									
Simulation Method	Beta GOS				HMM			DP	
	$\alpha_n = n, \beta_n = 1$	$\alpha_n = 5, \beta_n = 1$	$\alpha_n = 3, \beta_n = 1$	$\alpha_n = 1, \beta_n = 1$	$\alpha_n = 1, \beta_n = 3$	4 States	4 States	5 States	$\theta = 1$
	$\tau = 0.5$								
Nclusters Ground Truth	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18	4.97 ± 0.18
Nclusters Estimation	4.65 ± 1.20	4.55 ± 1.12	4.42 ± 1.01	4.26 ± 0.95	4.01 ± 0.86	3.92 ± 0.38	5.20 ± 1.82	5.20 ± 1.82	3.90 ± 1.56
Correct Assignment	0.88 ± 0.11	0.88 ± 0.11	0.88 ± 0.12	0.87 ± 0.12	0.84 ± 0.13	0.78 ± 0.11	0.85 ± 0.13	0.85 ± 0.13	0.77 ± 0.16
	$\tau = 1$								
Nclusters Ground Truth	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20	4.96 ± 0.20
Nclusters Estimation	4.28 ± 1.38	4.19 ± 1.32	3.97 ± 1.20	3.69 ± 1.05	3.31 ± 0.85	3.95 ± 0.32	4.36 ± 1.70	4.36 ± 1.70	3.90 ± 1.56
Correct Assignment	0.77 ± 0.13	0.78 ± 0.13	0.78 ± 0.13	0.75 ± 0.14	0.70 ± 0.14	0.77 ± 0.11	0.72 ± 0.15	0.72 ± 0.15	0.77 ± 0.16